

MATH 5A - FINAL EXAM  
Fall 2023

200 POINTS  
Instructions on Canvas.

---

- (1) Find the following limits if they exist (if the limit is  $\infty$  or  $-\infty$  say so). (12 points)

(a)  $\lim_{x \rightarrow 9} \frac{9-x}{\sqrt{x}-3}$  \_\_\_\_\_ (b)  $\lim_{x \rightarrow 5^+} \frac{-x^2}{x-5}$  \_\_\_\_\_

- 
- (2) If  $f(x) = \begin{cases} 5+x & \text{if } x > 2 \\ 15-x^3 & \text{if } x \leq 2 \end{cases}$  (6 points)

$\lim_{x \rightarrow 2^-} f(x)$  \_\_\_\_\_  
(a) Find  $\lim_{x \rightarrow 2^+} f(x)$  \_\_\_\_\_  
 $\lim_{x \rightarrow 2} f(x)$  \_\_\_\_\_  
(b) Is  $f(x)$  continuous for all real numbers? \_\_\_\_\_

---

- (3) Given  $f(x) = 2x^3 - 9x^2 + 12x - 3$  (14 points)

- (a) Find coordinates of local extrema (if any)  
(b) Find absolute extrema on  $[0, 3]$

Coordinates of Local Maximum(s): \_\_\_\_\_

Absolute Max Value: \_\_\_\_\_

Coordinates of Local Minimum(s): \_\_\_\_\_

Absolute Min Value: \_\_\_\_\_

(4) Find the derivative,  $\frac{dy}{dx}$  and simplify your answer: (30 points)

(a)  $y = \frac{x^5}{x^3 + 2x}$

(b)  $y = \tan(\cos(x^2))$

(c)  $y = \frac{3x}{\sqrt{x^2 - 4}}$

(5) Integrate:

(60 points)

(a)  $\int 4x\sqrt{x^2-5} dx$

(b)  $\int \frac{1}{\cos^2 x \sqrt{1+\tan x}} dx$

(c)  $\int_0^{\pi/3} \cos^4 x \sin x dx$

(d)  $\int \sec^2\left(\frac{x}{4}\right) dx$

(5 continued)

(d)  $\int x^4(x^2 - x^{3/2})dx$

(e)  $\int_{-2}^2 |x-1| dx$

---

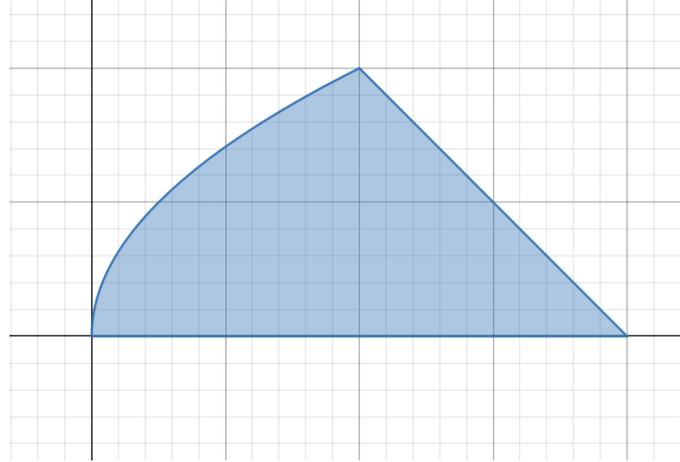
(6) Air is being pumped into a spherical balloon at a rate of  $5 \text{ cm}^3/\text{min}$ . Determine the rate at which the radius of the balloon is increasing when the diameter of the balloon is 3 cm. (give units)

( Volume of a sphere is  $\frac{4}{3}\pi r^3$  )

(12 points)

- (7) Consider the shaded region bound by  $y = \sqrt{x}$ ,  $y = 2 - x$ , and the x axis as shown. Set up (do not evaluate) integrals for each of the following situations. (25 points)

a) The area of the shaded region with respect to x



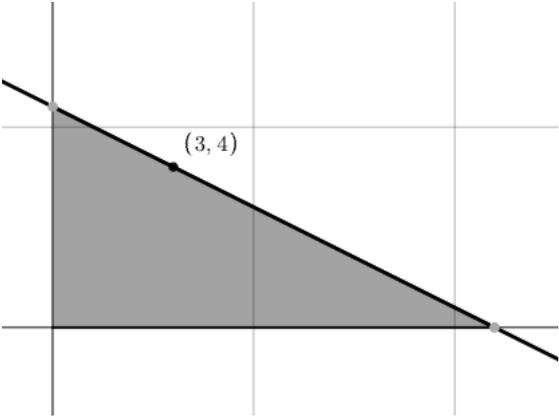
b) The area of the shaded region with respect to y

c) The volume of the solid generated when the region is revolved about the y axis, using disks/washers.

d) The volume of the solid generated when the region is revolved about the y axis, using cylindrical shells

e) The volume of the solid generated when the region is revolved about the line  $y = -1$  using either method.

(8) Find the equation of the line through (3,4) which cuts from the first quadrant a triangle of minimum area. Validate how you know you have found the *absolute* min. (12 points)

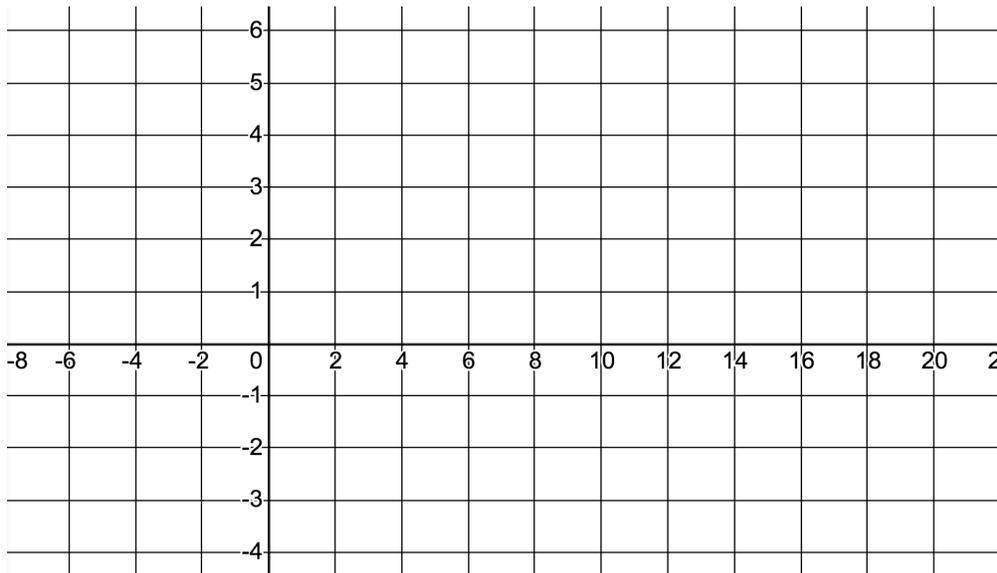


Equation of line : \_\_\_\_\_

(9) Graph and discuss the function. Show ALL work. Credit will not be given without supporting work and explanation. Be sure to graph x-intercepts, y-intercepts. Clearly show all information found in looking at the first and second derivative (including local extrema, inflection points, vertical and horizontal tangents and end behavior).

$$f(x) = x - 4\sqrt{x}$$

(17 points)



(10) Find the equation of the line tangent to the curve  $x^2 + 2xy + 4y^2 = 12$ , at the point  $(2,1)$ .